Quadrilaterals in the Coordinate Plane

Objective

Students will be able to find the coordinates of the vertices of special quadrilaterals placed in the coordinate plane.

Core Learning Goals

2.1.2 The student will identify and/or verify properties of geometric figures using the coordinate plane and concepts from algebra.

Materials Needed

Worksheets and graph paper – All graphs will be in the first quadrant. It would be helpful if the teacher had a graph on an overhead projector.

Pre-Requisite Concepts Needed

Students should be able to plot ordered pairs. All quadrilaterals in this lesson will be drawn in the first quadrant and will be labeled in alphabetical sequence, counter-clockwise. (HSA labels figures in alphabetical sequence, clockwise).

Approximate Time

One or two 45-minute lessons (Other quadrilaterals may be done as you study them.)

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Lesson plan – Squares and Parallelograms

Warm-Up/Opening Activity

Plot segment $\overline{AB}$. Given A (2, 0) and B (6, 0), draw a square in the first quadrant that has $\overline{AB}$ as one side.

The teacher should draw $\overline{AB}$ on the overhead sheet. After students have drawn their square, have them label the vertices so that they have square ABCD. They should also write the pair of coordinates near the appropriate vertex. Students can draw and label as the teacher graphs on the overhead sheet.

Answers: C (6, 4) and D (2, 4)

Development of Ideas

Key to the lesson: Today we are going to find the coordinates of missing vertices of squares and parallelograms. We will apply properties of quadrilaterals and what we know about graphing to find these vertices.

Let's try another square that is not as easy. Plot A (3, 5) and B (9, 5).

Now try to find the coordinates for C and D so that a square is formed.
Answers: C (9, 11) and D (3, 11)

After students have finished, discuss the relationships between the sides of the square. The length of $\overline{AB}$ is 6 so they know that the length of each side will be 6.

What is special about the coordinates of C? or what pattern do you see that can help you find C?

Assist students to see that the x value will be the same as the x value for B. The y value of C is computed by adding the length 6 to the y value of B.

What patterns can we see for point D?

Show:	D (3, 5 + 6) or (3, 11)	C (9, 5 + 6) or (9, 11)
	A (3, 5)	B (9, 5)

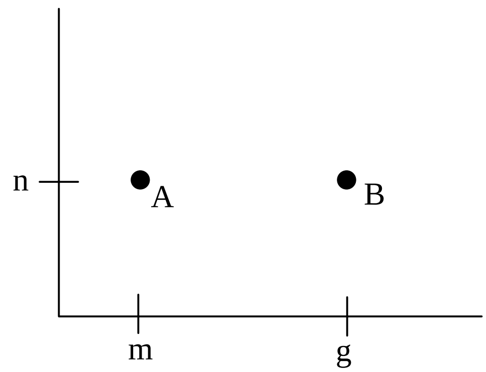
How do we know we have a square? Use mathematics to justify your answer.

Students should be able to tell you that you need to show 4 congruent sides as well as at least one right angle. You can then divide up the labor of using the distance formula and the slope formula. Side length = 6 and slopes of 2

Development of Ideas (Continued)

consecutive sides: 0 and undefined. Then have students write a justification, i.e., Because ABCD has 4 congruent sides and a right angle, we know from the theorems of parallelograms and by definition of a square that ABCD is a square.

Let's find the coordinates of the missing vertices when we are given variable coordinates. Given the coordinates A (m, n) and B (g, n), we can roughly plot these points. It is helpful to students to draw and label the axes with the variables as shown below:



What is the length of $\overline{AB}$? (g - m)

What are the coordinates of C and D? Give students time to work on this. Working in pairs might also be helpful.

Solution: C (g, n + (g - m)) and D (m, n + (g - m)).

Worksheet: **Finding Missing Vertices of Squares and Parallelograms**

Answers:

1. C (12,11) Students could apply any of the theorems that are used to prove a quadrilateral is a parallelogram. Such as: the slope of $\overline{AB}$ and $\overline{CD}$ is 0 so $\overline{AB}$ and $\overline{CD}$ are parallel since they have the same slope. Their lengths are congruent (7 by the distance formula). So ABCD is a parallelogram by the theorem: a quadrilateral is a parallelogram if one pair of opposite sides is parallel and congruent.
2. H (m + r – p, s)

Quadrilaterals in the Coordinate Plane

Development of Ideas (Continued)

Assignment

Answers:

1. B (9, 1) and C (9, 8)
2. C (5, 8) and D (1, 5) ABCD is a square because it is a parallelogram with 4 congruent sides (5, using the distance formula) and angle A is a right angle (slope of $\overline{AB}$ is 3/4 and the slope of $\overline{AC}$ is 4/-3).
3. B (e + h - f, f) and C (e + h - f, h)

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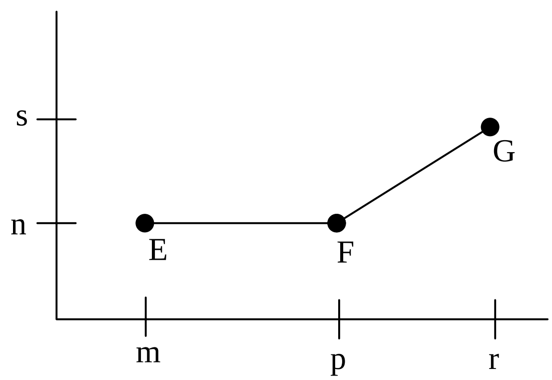
Finding Missing Vertices of Squares and Parallelograms

1. ABCD is a parallelogram. Given A (3, 4) B (10, 4) D (5, 11), plot and locate the point C. Label the parallelogram. What are the coordinates of C? Use mathematics to justify that your coordinates of C make ABCD a parallelogram.

2. EFGH is a parallelogram. Given E (m, n), F (p, n), and G (r, s). Locate H in the drawing. What are the coordinates of H?



Assignment

For square ABCD in the first quadrant, draw the square and find the missing vertices. What are the coordinates of these vertices?

1. A (2, 1) B( ) C( ) D (2, 8)

2. A (4, 1) B (8, 4) C( ) D ( )

   Use mathematics to justify that ABCD is a square.

3. A (e, f) B( ) C( ) D (e, h)

Quadrilaterals in the Coordinate Plane

Lesson Plan – Rectangles and Trapezoids

Warm-Up/Opening Activity

Plot and find the coordinates of the missing vertices of Rectangle ABCD.

A (1, 7), B(   ), C (3, 10), D(   )

The teacher should have plotted A and C on the overhead. After students have time to complete this activity, have a student explain how the vertices were determined.

Answer: B (3, 7) and D (1, 10)

Development of Ideas

Key to the lesson: Today we will continue our study of finding the coordinates of missing vertices of quadrilaterals. We will find the missing vertices of rectangles and trapezoids.

Worksheet: **Finding Missing Vertices of Rectangles and Trapezoids**

Answers:

1. C (n, j)

2. C (6, 5) Students should use the slope formula to show $\overline{AB}$ is parallel to $\overline{CD}$ (the slope of each segment is 0) and the distance formula to show $\overline{AD}$ is congruent to $\overline{BC}$, length is $\sqrt{13}$.) ABCD is an isosceles trapezoid by definition: a trapezoid is isosceles if the legs are congruent.

3. D (e + h - f, g)

Assignment

Answers:

1. C (10, 8) Students can use the slope formula to show: opposite sides are parallel (slopes are: 0 and undefined), then pairs of consecutive sides are perpendicular are perpendicular. ABCD is a parallelogram with right angles so ABCD is a rectangle by definition.

2. C (m, v)

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Development of Ideas (Continued)

Answers to the Assignment (Continued)

3. (8, 10) The results of finding the slopes of the four sides: $\overline{AB}$ and $\overline{CD}$ are parallel since m = 0 for each. The slope of $\overline{AD}$ is 2 and the slope of $\overline{BC}$ is -2. Therefore $\overline{AD}$ and $\overline{BC}$ must be the non-parallel sides: legs. Using the distance formula we find the length of each leg is the same: $\sqrt{20}$. By definition ABCD is an isosceles trapezoid because it is a quadrilateral with exactly one pair of parallel sides and the legs are congruent.

4. C (h - (n - w), q)

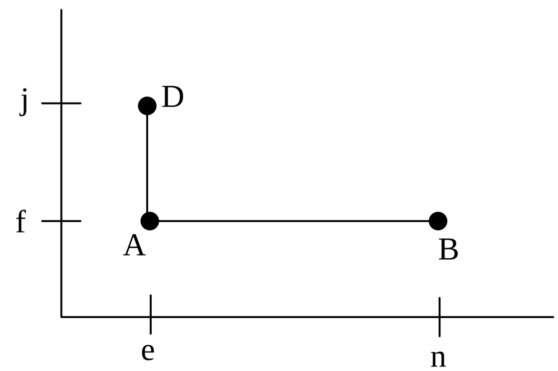
Closure

On a scale of 1 to 5 (5 meaning you could teach this to someone, 4 you feel like an expert but not quite ready to teach it. 3: you can do this with a little help. 2: you can get it started but still need lots of help and 1: you don’t know what you are doing!) please rate your knowledge of plotting and finding coordinates of quadrilaterals. (Check for understanding).

Quadrilaterals in the Coordinate Plane

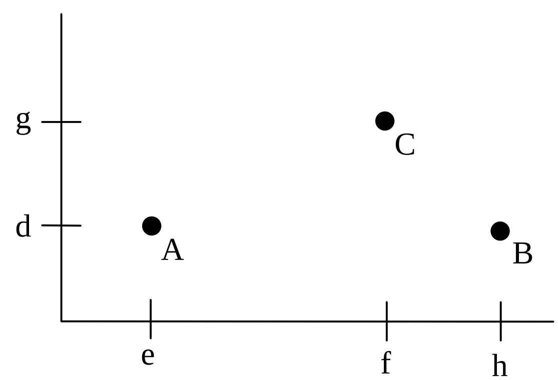
Finding Missing Vertices of Rectangles and Trapezoids

1. Find the coordinates of the missing vertex of rectangle ABCD.
   Given: A (e, f), B (n, f), D (e, j)



2. ABCD is an isosceles trapezoid in the first quadrant. Draw the trapezoid. What are the coordinates of C. Given: A (1, 2), B (8, 2), D (3, 5). Use mathematics to justify your answer.

3. Locate point D of an isosceles trapezoid in the first quadrant.
   Given: A (e, d), B (h, d), C (f, g). What are the coordinates of D?



Quadrilaterals in the Coordinate Plane

Finding Missing Vertices of Rectangles and Trapezoids (Continued)

Assignment

Find the coordinates of the missing vertices of the rectangles:

1. A (2, 3), B (10, 3), C (   ), and D (2, 8).
   Use mathematics to justify that ABCD is a rectangle.

2. A (p, q), B (m, q), C (   ),and D (p, v).

Find the coordinates of the missing vertices of the following isosceles trapezoids

3. A (3, 6), B (10, 6), C(   ), and D (5, 10).
   Use mathematics to justify that ABCD is an isosceles trapezoid.

4. A (w, t), B (h, t), C (   ), and D (n, q)